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Operating Leverage*

ROBERT NOVY-MARX
University of Chicago and NBER

Abstract. I derive and test implications of the operating leverage hypothesis for the cross-section of expected returns. Using a novel measure of operating leverage, I document that operating leverage predicts returns in the cross-section, and that strategies formed by sorting on operating leverage earn significant excess returns. Operating leverage also explains why the value premium is weak and non-monotonic across industries, but strong and monotonic within industries. Intra-industry differences in book-to-market are driven by differences in operating leverage, giving rise to expected return differences. Industry differences in book-to-market are driven by differences in the capital intensity of production unrelated to returns.

JEL Classification: E22, G12

1. Introduction

Theoretical models that generate a value premium generally rely on the "operating leverage hypothesis," introduced to the real options literature by Carlson et al. (2004). This hypothesis, expounded in some form at least as early as Lev (1974), states that production costs play much the same role as debt servicing in levering the exposure of a firms' assets to underlying economic risks. Operating leverage is critical to models that generate a value premium, because absent operating leverage growth options are riskier than deployed capital.

While operating leverage plays a critical role in these theories, there exists little supporting empirical evidence. Sagi and Seasholes (2007) and Gourio (2007) provide indirect evidence for the importance of operating leverage. Sagi and Seasholes show theoretically that operating leverage reduces asset return autocorrelation, and identify firm-specific attributes that improve the empirical performance of momentum strategies. Gourio presents evidence that operating income is more sensitive to gross domestic product shocks for value firms than for growth firms.

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I provide direct empirical evidence for the operating leverage hypothesis. I show, consistent with the hypothesis' explanation of the value premium, that firms with "levered" assets earn significantly higher average returns than firms with "unlevered" assets, where these characterizations refer to the level of operating (not financial) leverage.

The operating leverage hypothesis also predicts that the relation between expected returns and book-to-market should be weak and non-monotonic across industries, but strong and monotonic *within* industries. This prediction provides a theoretical basis for Cohen and Polk's (1998) contention that the value premium is largely an intra-industry phenomenon. The intuition for this result is as follows. "Value" firms have high book-to-markets because they are in capital intensive industries, in which case they have large book values relative to their market values, or because they are unprofitable, in which case they have low market values relative to their book values. The first type of value, essentially industry value, is not strongly related to expected returns. The second type of value, essentially intra-industry value, is strongly correlated with expected returns, because firms operating at low margins are more exposed to industry shocks than firms operating at high margins. A negative demand shock that reduces the price of the industry good one percent cuts the profitability of a producer operating at two percent margins in half, while only reducing the profitability of a producer operating at twenty percent margins by five percent. Because low margin producers are more exposed to economic risks, investors require a higher expected rate of return to hold these firms, and intra-industry value is thus strongly associated with higher expected returns.

Empirical investigation conducted here strongly supports these predictions. Sorting firms on the basis of intra-industry book-to-market generates significant variation in returns, while sorting firms on the basis of industry book-to-market fails to generate significant variation in returns, despite generating more variation in HML loadings than the intra-industry sort. As a consequence, the Fama-French three-factor model severely misprices the inter-industry value spread.

These results hold across both industry and intra-industry book-to-market quintiles. Value firms in value industries earn significantly higher returns than growth firms in value industries, and value firms in growth industries earn significantly higher returns than growth firms in growth industries. The converse is false. The returns to value firms in value industries are indistinguishable from the returns to value firms in growth industries, despite large differences in these firms' book-to-market ratios and HML loadings. Similarly, the returns to growth firms in value industries are indistinguishable from the returns to growth firms in growth industries.

These results suggest that a fundamental rethinking of the value premium is required. The value premium is *not* something that accrues to bricks-and-mortar. The data do not support contentions that "glamour" industries are "overpriced," and consequently provide low average returns going forward, and that value industries are "underpriced," and consequently provide high average returns going forward. In the data, the value premium accrues to inefficient producers. Efficient producers' large profit margins provide a cushion against negative economic shocks, and investors are willing to pay a premium for this "insurance." A return spread consequently arises between portfolios of high cost producers with low valuations and low cost producers with high valuations. It is intra-industry differences in firms, not industry characteristics, that drives the value premium.

The remainder of the paper is organized as follows. Section 2 discusses the basic intuition behind the operating leverage hypothesis, and develops testable predictions. Section 3 shows that high operating leverage firms generate higher average returns than low operating leverage firms. It also shows, consistent with the operating leverage hypothesis, that the relation between expected returns and book-to-market is weak across industries, but strong and monotonic within industries. Section 4 concludes.

2. Hypothesis Development

As with any real options model, a firm's value consists of two pieces: currently deployed asset and growth options, $V^i = V_A^i + V_G^i$ where i denotes the firm and the subscript A and G signify assets-in-place and growth options, respectively. The firm's expected excess returns depend on its exposure to the underlying risk factors. This exposure is a value weighted sum of the loadings of the firm's assets-in-place and the firm's growth options on these risks, i.e.,

$$\beta^i = \left(\frac{V_A^i}{V^i}\right)\beta_A^i + \left(\frac{V_G^i}{V^i}\right)\beta_G^i. \tag{1}$$

Just as the value of equity equals the value of assets minus the value of debt, the value of deployed assets consists of the capitalized value of the revenues they generate minus the capitalized cost of operating the assets, $V_A^i = V_R^i - V_C^i$. The exposure of the assets to the underlying risks is then a value weighted average of the exposures of the capitalized revenues and the capitalized operating costs,

$$\beta_A^i = \beta_R^i + \left(\frac{V_C^i}{V_A^i}\right)\left(\beta_R^i - \beta_C^i\right). \tag{2}$$

While growth options are almost always riskier than revenues from deployed capital in real options models, the presence of operating costs allows for deployed assets that are riskier than growth options. This is the operating leverage hypothesis of Carlson et al. (2004) and Sagi and Seasholes (2007). For operating leverage to significantly impact the riskiness of deployed capital requires both "highly geared assets" where gearing is defined as the ratio of capitalized operating costs to capitalized operating profits, $V_C^i/V_A^i \gg 0$, and "limited operational flexibility," $\beta_C^i \ll \beta_R^i$. For example, Zhang (2005) shows that increased operating leverage, in the form of higher fixed costs of production, leads to a higher value premium, employing an asymmetric capital adjustment cost function that is essentially designed to generate this limited operational flexibility (a quadratic adjustment penalty that is significantly higher for disinvestment than investment).

Highly geared assets tend to be those in firms with high levels of total operating costs (fixed plus variable), while low operational flexibility generally corresponds to fixed costs that represent a high proportion of total operating costs. High operating leverage, which is associated with both highly geared assets and low operating flexibility, is thus associated with high fixed costs of production, consistent with the common definition of operating leverage.

All firms basically satisfy the "highly geared" condition, with $V_C^i/V_A^i \gg 0$. Operating costs are generally an order of magnitude greater than operating profits, so V_C^i, the capitalized value of all future operating costs, should be large relative to V_A^i, the capitalized value of all future operating profits. While all firms basically satisfy the highly geared criterion, the level of gearing exhibits a great deal of variation in the cross-section, a fact that I will exploit in my empirical tests.

The second condition, "limited operational flexibility," is less obvious. Standard modeling devices used to generate tractability often do so precisely by generating equal cost and revenue betas, shutting down the operating leverage channel. For example, a production technology that is Cobb-Douglas with constant returns to scale in capital and non-capital factors of production results in cost and revenue betas that equate exactly.[1] Nevertheless, both the data and introspection seem to suggest that the existence of limited operational flexibility is not unreasonable. In response to negative shocks firms' revenues typically fall more quickly than they can reduce costs; prices are more responsive than firms' operations.

Combining equations (1) and (2) gives

$$\beta^i = \left(\frac{V_A^i}{V^i}\right)\left(\beta_R^i + \left(\frac{V_C^i}{V_A^i}\right)(\beta_R^i - \beta_C^i)\right) + \left(\frac{V_G^i}{V^i}\right)\beta_G^i. \tag{3}$$

I can simplify this equation by identifying assets-in-place with book assets and assuming that the capitalized cost of operating is proportional to annual operating costs. These heroic assumptions, which replace unobservable market value metrics with observable book value metrics, enable us to derive some basic empirical

[1] If instantaneous operating profits are given by $\pi =$ operating revenues $-$ operating costs $=$ $K^\alpha L^{1-\alpha} X - wL$, where K, L, w, and X are capital, labor, the wage rate, and demand, respectively, then maximizing over L yields operating revenues-over-operating costs equal to $1/(1-\alpha)$, which is constant. This implies the capitalized value of revenues and operating costs are in fixed proportion, and that their betas with respect to demand are identically equal.

predictions. They are not meant to be taken literally, though book-to-market is a good proxy for V_A^i/V^i if variation in book-to-market is driven primarily by difference in growth-options, not rents to deployed capital. Operating costs-to-assets is a good proxy for V_C^i/V_A^i if firms' positions in the cross-section of operating margins (operating profits-to-operating revenues) are persistent over time.

Under the assumptions discussed above, I can rewrite the previous equation as

$$\beta^i = BM^i \left(OL^i \left(\beta_R^i - \beta_C^i\right) - \left(\beta_G^i - \beta_R^i\right)\right) + \beta_G^i \tag{4}$$

where BM^i is firm i's book-to-market and OL^i is the firm's annual operating costs divided by book assets (multiplied by an arbitrary scale constant).

The previous equation provides a direct, testable hypothesis of the operating leverage hypothesis. In the equation book-to-market multiplies the difference in the risk factor loadings on deployed assets and growth options. The operating leverage hypothesis thus predicts that high book-to-market firms earn higher returns because they are relatively more exposed to assets-in-place, and assets-in-place are riskier than growth options. It also predicts that high operating leverage firms earn higher returns, because their assets-in-place are more levered (through operations), and thus riskier.

2.1 INDIRECT IMPLICATIONS

The operating leverage hypothesis predicts that firms with higher annual operating costs relative to their capital stocks should earn higher average returns. Closer inspection of Equation (4), on which this hypothesis is based, however, reveals limitations to direct inference on operating leverage. First, our empirical proxy for operating leverage, operating costs over book assets, is a better proxy for gearing (V_A^i/V^i) than it is for operating leverage ((V_A^i/V^i)($\beta_R^i - \beta_C^i$)), and thus implicitly assumes that the level of gearing and the degree of operational inflexibility are uncorrelated across firms. A more sophisticated analysis must recognize that higher operating costs may influence firms to reduce production sooner in the face of falling demand, resulting in higher cost betas for highly geared firms.

Moreover, the true level of gearing, which depends on the *capitalized* value of all future costs and revenues, is not truly observable. While market values provide a good proxy for the difference in the capitalized values of costs and revenues, it is difficult to find good proxies for these individually. Cross-industry differences in accounting practices, and the prevalence of leases, add further noise to accounting variables that might conceivably be related to operating leverage. Attenuation bias arising from noise in observed operating leverage reduces the power of tests that employ the measure. These facts provide incentives to develop testable indirect implications of the operating leverage hypothesis.

I consequently develop indirect implications explicitly in the appendix, using a dynamic model of operating leverage based on the industry equilibrium model of Novy-Marx (2009a). The model includes both elements necessary for generating cross-sectional variation in expected returns through the operating leverage channel: costly production and operational inflexibility. Costly production is introduced by assuming production utilizes non-capital factors of production (e.g., labor, raw materials), while operational inflexibility is introduced by assuming inflexibility in the factor mix (i.e., a clay-clay production technology) and that disinvestment is costly.

While the analysis of the model is somewhat complicated, the basic economic intuition driving the indirect implications of operating leverage is quite simple. "Value" firms can have high book-to-markets for two reasons, either because 1) they are in capital intensive industries, in which case they have large book values relative to their market values; or 2) because they are marginal producers operating at low margins, in which case they have low market values relative to their book values. The first type of value, essentially industry value, is not strongly related to expected returns, so variations in book-to-market due to variations in industry book-to-market are unpriced. The second type of value, essentially intra-industry value, is strongly correlated with expected returns. A firm operating at two percent margins is more exposed to industry shocks than a firm operating at twenty percent margins. The reason is that a negative demand shock that reduces the price of the industry good one percent cuts the profitability of the low margin producers in half, while only reducing the profitability of the high margin producer by five percent. Because the low margin producer is more exposed to economic risks, investors require a higher expected rate of return. As a consequence, intra-industry value is strongly associated with higher expected returns. The operating leverage hypothesis thus predicts that expected returns should be strongly correlated with book-to-market within an industry, but only weakly correlated across industries. That is, the model predicts that the value premium is an intra-industry phenomenon driven by inefficient producers, *not* something that accrues to industries that rely on bricks-and-mortar production.

Figure 1 depicts the model-implied relation between expected returns and book-to-market, both within and across industries. The bold, hump-shaped curve shows the relation between expected value-weighted average industry returns and industry book-to-market, i.e., the expected return / book-to-market relation *across* industries. Higher levels of operating costs are associated with lower industry book-to-markets, because rents that accrue to non-capital factors of production contribute to market values without contributing to book values. Higher levels of operating costs are not, however, strongly associated with an industry's expected returns. Higher industry operating costs increase the gearing of deployed capital, but also increase the operational flexibility of capital as firms are more willing to shut down unprofitable production to avoid the high flow costs associated with production. The net impact

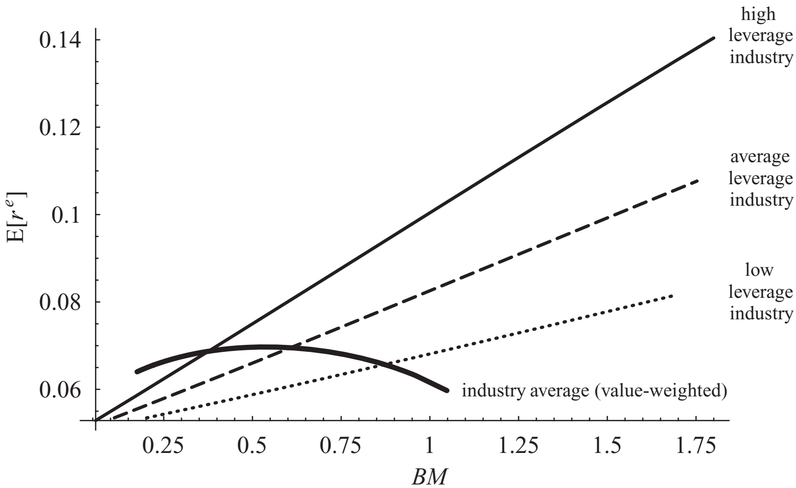

Figure 1. The relation between book-to-market and expected returns within and across industries
This figure depicts the unconditional relation between expected excess returns and book-to-market within three different industries, and also across industries. The top line (solid) shows the expected return / book-to-market relation in a high operating leverage industry (operating costs-to-assets equal to two and a half), the middle line (dashed) shows a moderate operating leverage industry (operating costs-to-assets equal to one), while the bottom line (dotted) shows a low operating leverage industry (operating costs-to-assets equal to two fifths). The bold curve depicts the relation between expected excess industry returns and industry book-to-market.

on operating leverage is indeterminate, resulting in a weak, non-monotonic inter-industry relation between book-to-market and expected returns.

The upward sloping lines in Figure 1 show the relation between expected returns and book-to-markets *within* industries. The top, solid line depicts a high operating leverage (e.g., labor intensive) industry, the middle, dashed line an average industry, and the bottom, dotted line a low operating leverage (e.g., capital intensive) industry. Within industries the relation between expected returns and book-to-market is strong and monotonic. Within industries inefficient firms generate less profits and are more exposed to economic shocks, and consequently have higher book-to-markets and earn higher returns than more efficient, lower book-to-market firms.

Together the predictions that variations in book-to-market within an industry are strongly associated with variation in expected returns, while variations in industry book-to-market are not, represent a simple, testable hypothesis of the operating leverage hypothesis that does not require a direct measure of operating leverage. Moreover, inspection of Figure 1 reveals a more subtle prediction, that the relation between book-to-market and expected returns is stronger in growth industries than it is in value industries.

3. Empirical Evidence

To test the simple prediction that high operating leverage firms generate higher returns than low operating leverage firms, I first run Fama-MacBeth regressions employing operating leverage, defined as annual operating costs divided by assets (Compustat item AT), where operating costs is cost of goods sold (COGS) plus selling, general, and administrative expenses (XSGA). Scaling operating costs by the market value of assets would result in a closer proxy for the operating leverage measure (V_C/V_A) provided in Equation (3), because the market value of a firm's assets provides a better proxy for the market value of its currently deployed capital than does book assets. This market-based measure is less interesting empirically, however, because of its high correlation with book-to-market. It should be easier to identify the impact of operating leverage that is distinct from value effects using the book-based measure.

These Fama-MacBeth regressions include controls for book-to-market, size, and performance over the prior month and year, and exclude banks and financial firms (i.e., firms with a one-digit SIC code of six).[2] The book-to-market and operating leverage measures are updated at the end of each June, using accounting data from the fiscal year ending in the previous calendar year.

Finally, while the theory concerns firms' assets, I test the predictions using equity returns. The reason is that equity returns are readily available, while asset returns (and the debt returns and debt values that could be used to construct them) are not. Using equity data biases the tests *against* rejecting the irrelevance of operating leverage. Operating leverage and financial leverage are negatively correlated in the data: the annual average correlation between operating leverage and debt, defined as long term debt (Compustat DLTT) plus debt in current liabilities (DLC) scaled by assets (AT), is negative three percent. Consequently, even if higher operating leverage is truly associated with higher asset returns, the higher financial leverage of firms with low operating leverage could result in low operating leverage firms generating higher average equity returns.

Table I provides results of the Fama-MacBeth regressions of firms' returns on operating leverage and different sets of controls, over the sample spanning

[2] Book-to-market is book equity scaled by market equity, where market equity is lagged six months to avoid taking unintentional positions in momentum. Book equity is shareholder equity, plus deferred taxes, minus preferred stock, when available. For the components of shareholder equity, I employ tiered definitions largely consistent with those used by Fama and French (1993) to construct HML. Stockholders equity is as given in Compustat (SEQ) if available, or else common equity plus the carrying value of preferred stock (CEQ + PSTX) if available, or else total assets minus total liabilities (AT − LT). Deferred taxes is deferred taxes and investment tax credits (TXDITC) if available, or else deferred taxes and/or investment tax credit (TXDB and/or ITCB). Prefered stock is redemption value (PSTKR) if available, or else liquidating value (PSTKRL) if available, or else carrying value (PSTK).

Table I. Fama-MacBeth regressions employing operating leverage

This table reports results from Fama-MacBeth regressions of firms' returns on operating leverage (OL), defined as cost of goods sold (Compustat annual data item COGS) and selling, general and administrative expenses (XSGA) scaled by assets (AT). Regressions include controls for book-to-market (log(bm)), size (log(me)), and past performance measured at horizons of one month ($r_{1,0}$) and twelve to two months ($r_{12,2}$). The sample covers January 1963 to December 2008, and excludes financial firms (those with one digit SIC codes equal to six).

Independent variables	Slope coefficients ($\times 10^2$) and [test-statistics] from regressions of the form $r_{tj} = \beta' \mathbf{x}_{tj} + \epsilon_{tj}$							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
OL	0.11	0.10			0.13	0.04		0.08
	[3.31]	[3.36]			[4.02]	[1.56]		[2.52]
log(BM)			0.35		0.35		0.26	0.25
			[7.34]		[7.33]		[4.97]	[4.91]
log(ME)				−0.16		−0.16	−0.13	−0.13
				[−4.00]		[−3.95]	[−3.13]	[−2.99]
$r_{1,0}$		−4.98	−5.07	−5.14	−5.15	−5.19	−5.37	−5.43
		[−12.1]	[−12.5]	[−13.4]	[−12.8]	[−13.6]	[−14.4]	[−14.7]
$r_{12,2}$		0.73	0.70	0.77	0.68	0.74	0.71	0.69
		[4.88]	[4.53]	[5.60]	[4.43]	[5.46]	[5.31]	[5.16]

January 1963 to December 2008. The first two specifications demonstrate that the operating leverage measure has significant power predicting returns, either alone or with past performance controls. The third and fourth specifications show the standard results that both book-to-market and size also have significant power predicting returns. The coefficient on operating leverage has roughly the same magnitude and significance as that on size, and half that on book-to-market. Specifications (5) to (7) show that including both operating leverage and book-to-market as explanatory variables has essentially no effect on the magnitude or significance of the coefficient estimates on either variable, but including size as an explanatory variable weakens the roles of both operating leverage and book-to-market. The final specification shows that all three variables have significant explanatory power when employed jointly. That is, operating leverage helps predict returns even after controlling for size, book-to-market and past performance. Dropping the past performance controls has no material impact on these results.

3.1 PORTFOLIO SORTS ON OPERATING LEVERAGE

The Fama-MacBeth regressions show that operating leverage has power predicting stock returns. This section shows this by analyzing the returns to portfolios sorted on the basis of operating leverage. Portfolios are constructed employing a quintile sort, using New York Stock Exchange (NYSE) break points, and reformed at the end of each June. Table II shows time-series average characteristics of these portfolios.

Table II. Operating leverage portfolio summary statistics

The table reports time-series average characteristics of portfolios sorted on operating leverage, defined as cost of goods sold (COGS) plus selling, general and administrative expenses (XSGA), scaled by the book value of assets (AT). Investment-to-assets is the annual change in total property, plant and equipment (PPEGT) and inventories (INVT) scaled by assets. Return-on-assets is quarterly income before extraordinary items (QIB) scaled by assets. Summary statistics are for the period January 1963 to December 2008, except for investment-to-assets and return-on-assets, which are only available for the period January 1972 to December 2008.

	Operating leverage portfolio				
Characteristic	(Low)	(2)	(3)	(4)	(High)
Operating leverage	0.14	0.49	0.84	1.18	2.18
Average capitalization ($\$10^6$)	816	1,629	1,199	832	467
Book-to-market	1.02	0.70	0.78	0.64	0.62
Book-to-market/industry book-to-market	1.12	0.99	1.03	0.98	0.97
Investment-to-assets	10.9%	8.1%	6.7%	5.3%	6.9%
Return-on-assets (annualized)	2.2%	5.1%	7.1%	6.3%	4.7%
Number of firms	431	742	888	985	1,258

Table III provides return characteristics of the portfolios sorted on operating leverage. The portfolios' average monthly excess returns are given, both value and equal weighted, together with the factor loadings and alphas from time-series regressions of the portfolios' returns on both the Fama-French and Chen-Zhang factors.

The left half of the table shows that the levered portfolio yields significantly higher returns than the unlevered portfolio, even though expected correlation between operating costs and operational flexibility should bias the operating leverage sort against generating significant variation in expected returns. Over the sample period, January 1963 to December 2008, the levered portfolio earned 44 basis points per month more than the unlevered portfolio on a value-weighted basis, with a test-statistic equal to 2.69, and 51 basis points per month more on an equal-weighted basis, with a test-statistic equal to 3.35. The realized annual Sharpe ratios of the levered-minus-unlevered strategies are 0.40 and 0.50, value and equal weighted respectively, similar to that of the value-weighted high-minus-low quintile book-to-market strategy over the same period (0.43). The levered-minus-unlevered strategies also have significant three-factor alphas, though moderate SMB loadings, smaller than that obtained sorting on book-to-market, explains some of the strategies' returns. This is consistent with the fact, from Table I, that controlling for size marginally reduces the explanatory power of operating leverage (or book-to-market). In four-factor regressions none of the portfolios loads significantly on UMD.

The right half of the table shows results of time-series regression of the levered-minus-unlevered strategies' returns on the Chen-Zhang (2010) factors over the

Table III. Excess returns and alphas to portfolios sorted on operating leverage

This table shows monthly average excess returns to portfolios sorted on operating leverage, and results of time-series regressions of these portfolios' returns on the both the Fama-French factors and the Chen-Zhang factors. Operating leverage is cost of goods sold (COGS) plus selling, general and administrative expenses (XSGA), scaled by the book value of assets (AT).

	January 1963 – December 2008 Fama-French model					January 1972 – December 2008 Chen-Zhang model				
	r^e	α	MKT	SMB	HML	r^e	α	MKT	INV	ROA
Panel A: Value-weighted results										
Operating leverage quintiles										
Low	0.12	−0.25	0.92	−0.03	0.06	0.16	0.29	0.82	−0.30	0.32
	[0.55]	[−1.99]	[30.5]	[−0.84]	[1.25]	[0.63]	[2.10]	[27.1]	[−4.27]	[−9.55]
2	0.34	0.07	0.98	−0.10	−0.17	0.31	0.03	0.98	−0.04	−0.07
	[1.71]	[1.21]	[72.0]	[−5.45]	[−8.13]	[1.35]	[0.42]	[61.4]	[−0.98]	[−3.94]
3	0.46	0.10	1.03	0.03	−0.07	0.40	0.04	1.03	−0.05	−0.01
	[2.24]	[1.90]	[83.7]	[1.57]	[−3.52]	[1.71]	[0.76]	[80.7]	[−1.65]	[−0.48]
4	0.43	0.03	1.02	0.06	0.01	0.43	0.01	1.03	−0.07	0.06
	[2.11]	[0.54]	[79.7]	[3.22]	[0.28]	[1.81]	[0.15]	[77.6]	[−2.27]	[3.91]
High	0.56	0.11	0.99	0.23	0.03	0.53	0.10	1.02	−0.08	0.08
	[2.61]	[1.45]	[52.6]	[9.28]	[1.17]	[2.20]	[0.96]	[46.8]	[−1.47]	[3.48]
H-L	0.44	0.36	0.07	0.27	−0.02	0.38	−0.19	0.20	0.23	0.40
	[2.69]	[2.21]	[1.82]	[5.08]	[−0.40]	[1.99]	[−1.05]	[5.05]	[2.40]	[9.02]
Panel B: Equal-weighted results										
Operating leverage quintiles										
Low	0.33	−0.27	0.97	0.51	0.25	0.31	0.46	0.91	−0.20	−0.41
	[1.37]	[−2.11]	[31.5]	[12.6]	[5.31]	[1.07]	[2.82]	[25.7]	[−2.47]	[−10.5]
2	0.52	−0.11	1.05	0.76	0.10	0.45	0.53	1.07	−0.04	−0.46
	[1.98]	[−1.25]	[50.2]	[27.3]	[3.14]	[1.50]	[3.91]	[35.9]	[−0.57]	[−14.0]
3	0.70	0.03	1.07	0.93	0.12	0.66	0.67	1.13	0.07	−0.44
	[2.56]	[0.35]	[56.4]	[36.6]	[4.11]	[2.11]	[4.39]	[33.9]	[0.84]	[−12.0]
4	0.73	0.00	1.03	0.98	0.23	0.70	0.60	1.10	0.23	−0.38
	[2.70]	[0.06]	[52.6]	[37.3]	[7.85]	[2.30]	[3.59]	[30.2]	[2.69]	[−9.49]
High	0.84	0.08	0.99	1.05	0.31	0.78	0.67	1.06	0.26	−0.37
	[3.09]	[0.86]	[43.1]	[34.1]	[8.84]	[2.55]	[3.62]	[26.0]	[2.74]	[−8.33]
H-L	0.51	0.35	0.03	0.54	0.06	0.48	0.22	0.15	0.46	0.04
	[3.38]	[2.53]	[0.82]	[12.1]	[1.23]	[2.77]	[1.20]	[3.68]	[5.06]	[0.83]

period for which they are available, January 1972 to December 2008. The Chen-Zhang factors, which are motivated by a simple investment-based asset pricing model, do a good job pricing the levered-minus-unlevered spread, primarily because of the variation in the operating leverage portfolios' loadings on the model's investment factor (INV). High operating leverage firms invest less than low operating leverage firms, especially on an equal-weighted basis. As a result, neither the value nor equal weighted spread is significant relative to the Chen-Zhang factors.

The Chen-Zhang model performs poorly, however, explaining the underlying equal-weighted operating leverage portfolios. These portfolios all have relatively large, negative loadings on the model's productivity factor (ROA), but don't generate particularly low returns. ROA generates extremely large returns over the sample (1.05 percent per month), so these negative ROA loadings result in large positive Chen-Zhang alphas (an effect that is absent from the levered-minus-unlevered spread, where the ROA loadings on the long and short side essentially cancel). As a result, a GRS (Gibbons et al. (1989)) test emphatically rejects the hypothesis that the true Chen-Zhang pricing errors on the five portfolios are jointly zero ($F_{5,430} = 4.75$, p-value $= 0.0\%$). A similar test fails to reject the hypothesis that the true Fama-French pricing errors are jointly zero over the same period ($F_{5,430} = 1.82$, p-value $=$ 10.8%).

The Chen-Zhang model's mispricing of the equal-weighted operating leverage portfolios can be partly attributed to the fact that the model, as noted in Chen and Zhang (2010), tends to exacerbate, not explain, the size premium. Large firms tend to have higher ROA loadings than higher return small firms, and as a result SMB had a Chen-Zhang alpha of 38 basis points per month over the sample period, with a test-statistic of 2.57.[3] The operating leverage portfolios, because they are equal-weighted, carry large SMB loadings.

3.2 THE VALUE PREMIUM WITHIN AND ACROSS INDUSTRIES

The model predicts that book-to-market and expected returns are strongly correlated *within* an industry, but that the relation between book-to-market and expected returns is weak, and non-monotonic, *across* industries. If this is truly the case, then Fama-MacBeth regressions of stocks' returns on a decomposition of log book-to-market into industry and intra-industry components should yield a large, significant coefficient estimate on the intra-industry component, and a small, insignificant estimate on the industry component. If log book-to-market is the true explanatory variable, however, then Fama-MacBeth regressions of stocks' returns on a decomposition of log book-to-market into industry and intra-industry components should yield identical coefficient estimates on the different components.

Table IV tests these competing hypotheses over the sample covering July 1926 through December 2008, employing the Davis et al. (2000) book equity data prior to the availability of Compustat data. The tests decompose log book-to-market into industry and intra-industry components using three different methodologies. The first set of tests uses the log of book-to-market scaled by industry as the intra-industry value measure and log industry book-to-market as the industry value measure. The second set of tests uses log book-to-market demeaned by industry

[3] SMB generated 24 basis points per month, with a test-statistic of 1.78, over the same period.

Table IV. Fama-MacBeth regressions employing measures of value within and across industries, July 1926 to December 2008

This table reports results from Fama-MacBeth regressions of firms' returns on three different pairs of intra-industry and industry value measures: 1) log book-to-market relative to industry book-to-market and log industry book-to-market; 2) log book-to-market demeaned by industry and industry average book-to-market; and 3) book-to-market ranking (percentile) within industry and industry book-to-market ranking. Regressions include controls for size (log(me)), prior month's performance ($r_{1,0}$), and prior years's performance ($r_{12,2}$).

	Slope coefficients ($\times 10^2$) and [test-statistics] from regressions of the form $r_{tj} = \beta' \mathbf{x}_{tj} + \epsilon_{tj}$ intra- and inter-industry value measures									
		Log industry-relative book-to-market and log industry book-to-market			Log book-to-market demeaned by industry and industry average log book-to-market			Book-to-market percentile within industry and industry book-to-market percentile		
Independent variables	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Log book-to-market	0.25									
	[5.23]									
Intra-industry value		0.24		0.27	0.27		0.28	0.75		0.76
		[6.93]		[6.21]	[6.96]		[6.69]	[7.51]		[7.39]
Industry value			0.05	0.15		0.05	0.07		0.14	0.16
			[0.76]	[2.00]		[0.54]	[0.71]		[1.09]	[1.30]
log(ME)	−0.14	−0.14	−0.18	−0.13	−0.14	−0.17	−0.13	−0.14	−0.18	−0.13
	[−3.91]	[−3.82]	[−4.45]	[−3.67]	[−3.75]	[−4.36]	[−3.67]	[−3.79]	[−4.45]	[−3.75]
$r_{1,0}$	−7.05	−6.91	−7.14	−7.19	−6.95	−7.07	−7.24	−6.92	−7.11	−7.10
	[−20.0]	[−19.2]	[−19.7]	[−20.4]	[−19.3]	[−19.7]	[−20.7]	[−19.2]	[−19.6]	[−20.0]
$r_{12,2}$	0.71	0.72	0.80	0.70	0.72	0.77	0.69	0.73	0.80	0.72
	[3.54]	[3.52]	[4.30]	[3.52]	[3.55]	[4.15]	[3.47]	[3.61]	[4.32]	[3.63]
Intra-minus-inter difference				0.11			0.21			0.60
				[2.11]			[2.61]			[6.42]

as the intra-industry value measure and industry average log book-to-market as the industry value measure. The last set of tests uses firms' book-to-market rankings within their industries as the intra-industry value measure and the ranking of the book-to-market of firm's industry as the industry value measure, where these rankings are parameterized between zero and one.

Specification (1) shows the standard result, that log book-to-market has significant explanatory power predicting stock returns in Fama-MacBeth regressions, even after controlling for size (log market equity), short run reversals (prior month's performance) and momentum (performance over the first eleven months of the preceding year).

Specifications (2)–(4) test the roles of intra-industry and industry measures of book-to-market, employing the explanatory variables log book-to-market relative to industry book-to-market and log industry book-to-market, respectively. Specification (2) shows that the coefficient estimate on log industry-relative book-to-market is roughly as large as that on log book-to-market in specification (1), but estimated more precisely. Specification (2) shows that log industry book-to-market has no explanatory power on its own. Specification (4) shows that the intra-industry measure has significantly more power than the industry measure. In regressions that include both measures, the coefficient estimate on log industry-relative book-to-market is nearly twice that on log industry book-to-market, and the difference is statistically significant.

Specifications (5)–(7) repeat the test of specifications (2)–(4), employing log book-to-market demeaned by industry as the intra-industry value measure, and industry average log book-to-market as the industry value measure. Again, the coefficient on the intra-industry measure is large and highly significant (specification (5)), while the coefficient on the industry measure is small and insignificant (specification (6)), and the difference between the two coefficient estimates is large and significant (specification (7)).

Specifications (8)–(10) repeat the test of specifications (2)–(4) and (5)–(7), employing the book-to-market ranking within a firm's industry, parameterized from zero to one, as the intra-industry value measure, and industry book-to-market ranking, parameterized similarly, as the industry value measure. The coefficients on these variables can thus be interpreted as the difference in expected monthly average returns between the highest and lowest book-to-market firms within a given industry, and the difference in expected monthly average returns between firms in the highest and lowest book-to-market industries, respectively. These results are again consistent with the earlier specifications. The coefficient on the intra-industry measure is large and highly significant (specification (8)), while the coefficient on the industry measure is small and insignificant (specification (9)), and the difference between the two coefficient estimates is large and significant (specification (10)).

These results confirm the basic predictions of Figure 1, that the relation between expected returns and book-to-market is strong within industries, but weak across industries. Further inspection of Figure 1, however, reveals deeper, more nuanced predictions. Not only does the model predict that expected returns are increasing with book-to-market within industries, it predicts that this relation is stronger in growth industries. That is, the model predicts that the slope of expected returns on a firm's book-to-market is decreasing in the book-to-market of the firm's industry.

Fama-MacBeth regressions also confirm this prediction. The estimated relation between a firm's excess monthly returns (in percent per month), its book-to-market, and the book-to-market of its industry, is given by

$$\mathbf{E}[r_{ij} - r^{\mathrm{mkt}}] = \underset{[-0.71]}{0.19} + \left(\underset{[5.98]}{0.33} - \underset{[-1.99]}{0.16}\ BM^{i}\right) \log\left(\frac{BM^{ij}}{BM^{i}}\right) + \underset{[2.13]}{0.16} \log(BM^{i}) \quad (5)$$

where an index i denotes industry i and an index ij denotes firm j in industry i, and the numbers in the square brackets are the test-statistics of the coefficient estimates. The slope on log industry-relative book-to-market is significantly decreasing with industry book-to-market, implying that the relation between firms' expected returns and book-to-market ratios is stronger in low book-to-market (growth) industries than it is in high book-to-market (value) industries.

This result makes intuitive sense in the context of my model. The value premium reflects differences in returns expected to accrue to inefficient and efficient firms. Relatively inefficient firms are more exposed to economic risks, and have relatively lower book-to-market ratios, than more efficient firms. Relative differences in book-to-market correspond to smaller absolute differences in low book-to-market industries, generating a stronger relation between book-to-market levels and expected returns.

3.2.1 Portfolio Sorts on Intra-Industry Book-to-Market and Industry Book-to-Market

The preceding results suggest that "value" has both a priced and an unpriced component. The priced component appears to be related to variation in firms' efficiencies, identifiable as differences in book-to-market ratios within industries. The unpriced component appears to be related to industry variation, which affects book-to-market ratios but is largely unrelated to differences in expected returns.

In order to test this prediction, I perform separate sorts based on intra-industry book-to-market and industry book-to-market. The first sort is used to identify value (inefficient) and growth (efficient) firms within industries, while the second sort is used to generate value and growth industries.

Table V. Summary statistics for portfolios sorted on book-to-market in and across industries

This table reports time-series average characteristics of portfolios sorted on book-to-market within industries (panel A) and industry book-to-market (panel B). The sample covers July 1926 through December 2008.

Characteristic	(Low)	(2)	(3)	(4)	(High)
Panel A: intra-industry book-to-market portfolios					
Book-to-market	0.39	0.69	1.03	1.42	3.52
Average capitalization ($\$10^6$)	1,120	799	512	333	173
Number of firms	632	504	511	567	615
Panel B: industry book-to-market portfolios					
Book-to-market	0.39	0.60	0.79	1.03	1.88
Average capitalization ($\$10^6$)	625	474	440	471	585
Number of firms	707	718	670	759	851

The intra-industry sort each year assigns each stock to a portfolio based on the firm's book-to-market ratio relative to other firms in the same industry. For example, a firm is assigned to the value portfolio if it has a book-to-market ratio higher than eighty percent of NYSE firms in the same industry. Each quintile portfolio consequently contains roughly twenty percent of the firms in each industry. The industry sort each year assigns each stock to a quintile portfolio based on the book-to-market of the firm's industry (total industry book value divided by total industry market value). Industries are the 49 defined by Fama and French (1997). The sample again covers July 1926 through December 2008.

Table V gives time-series average characteristics of the portfolios sorted on intra-industry book-to-market (panel A) and industry book-to-market (panel B). The table shows that the dispersion in book-to-market within industries is approximately twice that observed across industries. It is also interesting to note the manner in which firm size varies across book-to-market portfolios for the two different sorts. While size is negatively correlated with intra-industry book-to-market, in much the same way that it is with straight book-to-market, it is essentially uncorrelated with industry book-to-market. That is, while value firms within an industry tend to be significantly smaller than growth firms in the same industry, firms in value industries are roughly as large as firms in growth industries. This result is consistent with the model employed in this paper, and helps explain why the value effect is concentrated in small firms. Size helps distinguish value firms that generate higher returns because they are truly more exposed to risk from value firms that are average producers, with average risk exposures, in high book-to-market industries.

Table VI reports the portfolios' average excess returns, and results of time series regressions of the portfolios' returns on the Fama-French factors. Value-weighted results are presented on the left half of the table, with equal-weighted results on the right. Panel A shows that the intra-industry book-to-market sort generates

Table VI. Excess returns and alphas to portfolios sorted on book-to-market in and across industries

This table shows monthly average excess returns to portfolios sorted on 1) book-to-market within industries and 2) industry book-to-market, and results of time-series regressions of these portfolios' returns on the Fama-French factors. The sample covers July 1926 to December 2008.

	Value-weighted results					Equal-weighted results				
	r^e	α	MKT	SMB	HML	r^e	α	MKT	SMB	HML
					Panel A: Intra-industry portfolios					
Intra-industry BM quintiles										
Low	0.52	0.01	1.03	−0.07	−0.18	0.48	−0.21	1.05	0.57	−0.14
	[2.94]	[0.41]	[191]	[−8.49]	[−23.0]	[2.23]	[−3.74]	[96.8]	[32.7]	[−9.12]
2	0.60	0.02	0.97	−0.02	0.04	0.75	−0.04	1.03	0.57	0.14
	[3.43]	[0.61]	[170]	[−1.99]	[4.39]	[3.46]	[−0.87]	[117]	[41.0]	[10.6]
3	0.68	−0.03	1.03	0.01	0.27	0.91	−0.00	1.02	0.71	−0.35
	[3.54]	[−0.64]	[137]	[0.53]	[24.8]	[3.91]	[−0.03]	[129]	[55.6]	[30.7]
4	0.74	0.03	0.98	0.11	0.28	1.10	0.06	1.04	0.89	0.54
	[3.85]	[0.63]	[93.2]	[6.36]	[18.3]	[4.29]	[1.29]	[112]	[59.8]	[40.4]
High	0.87	0.07	1.04	0.23	0.36	1.43	0.24	1.05	1.13	0.75
	[4.17]	[1.46]	[112]	[15.3]	[27.0]	[2.20]	[0.96]	[46.8]	[−1.47]	[3.48]
H-L	0.35	0.06	0.01	0.30	0.54	0.95	0.45	0.01	0.56	0.89
	[3.68]	[0.92]	[0.60]	[15.3]	[30.4]	[6.62]	[6.23]	[0.32]	[25.0]	[43.8]
					Panel B: Industry portfolios					
Industry BM quintiles										
Low	0.52	0.01	1.06	−0.02	−0.28	0.83	−0.01	1.12	0.89	−0.07
	[2.77]	[0.24]	[104]	[−1.02]	[−19.0]	[3.31]	[−0.15]	[81.5]	[40.7]	[−3.53]
2	0.58	0.04	0.99	0.07	−0.15	0.82	−0.08	1.05	0.84	0.20
	[3.20]	[0.69]	[94.7]	[3.89]	[−9.73]	[3.37]	[−1.25]	[81.7]	[40.7]	[10.9]
3	0.64	0.07	0.98	−0.02	−0.01	0.93	0.05	0.98	0.80	0.30
	[3.56]	[1.16]	[82.4]	[−0.88]	[−0.70]	[4.03]	[0.72]	[77.1]	[39.3]	[16.2]
4	0.73	0.08	0.94	−0.01	0.22	0.97	0.04	0.99	0.67	0.47
	[4.00]	[1.29]	[76.2]	[−0.46]	[12.3]	[4.15]	[0.52]	[76.1]	[32.09]	[24.7]
High	0.67	−0.16	0.99	0.01	0.60	0.99	−0.11	0.99	0.81	0.81
	[3.26]	[−2.83]	[88.6]	[0.66]	[37.1]	[3.84]	[−1.74]	[78.9]	[40.3]	[44.6]
H-L	0.15	−0.17	−0.07	0.03	0.88	0.17	−0.10	−0.13	−0.08	0.88
	[1.23]	[−2.21]	[−4.71]	[1.16]	[39.6]	[1.29]	[−1.17]	[−7.63]	[−3.10]	[36.1]

significant variation in the portfolios average returns. The Sharpe ratios on intra-industry value spread (0.41 value-weighted and 0.74 equal-weighted) exceed those generated by a straight book-to-market sort (0.31 and 0.61).

The Fama-French factors accurately price the value-weighted intra-industry book-to-market portfolios. While the observed market model root-mean-squared pricing error on the five portfolios is fifteen basis points per month, and a GRS test strongly rejects the hypothesis that the market model pricing errors are jointly zero ($F_{5,984} = 2.95$, for a p-value $= 1.2\%$), the observed three factor model root-mean-squared pricing error is only four basis points per month, and a GRS test fails to reject the hypothesis that the three-factor pricing errors are jointly zero

($F_{5,982} = 1.34$, for a p-value $= 24.4\%$). The Fama-French factors cannot price the equal-weighted intra-industry book-to-market portfolios. The equal-weighted intra-industry value spread generates 45 basis points per month relative to the Fama-French model, and the test-statistic on these abnormal returns exceeds six.

The inter-industry results, presented in Panel B, contrast strongly with the intra-industry results presented in Panel A. Value industries do not provide significantly higher returns than growth industries, despite having significantly higher book-to-market ratios and HML loadings. This fact is difficult to reconcile with Lettau and Wachter's (2007) duration-based explanation of the value premium. Industry market-to-book has significant power predicting industry revenue growth over the succeeding five years. Slow growing value industries have cash flows weighted more towards the present, while fast growing growth industries' have cash flows weighted more towards the future. The fact that low-duration value industries do not significantly outperform high-duration growth industries is contrary to the predictions of the Lettau-Wachter model.

The three factor model also performs worse than the market model in explaining the value-weighted returns to portfolios sorted on industry book-to-market. While the observed market model root-mean-squared pricing error on the five inter-industry book-to-market portfolios is nine basis points per month, and a GRS test fails to reject the hypothesis that the market model pricing errors are jointly zero ($F_{5,984} = 1.46$, for a p-value $= 20.2\%$), the observed three factor model root-mean-squared pricing error is eleven basis points per month, and a GRS test weakly rejects the hypothesis that the three-factor pricing errors are jointly zero ($F_{5,982} = 2.19$, for a p-value $= 5.4\%$). The Fama-French factors do a good job, however, pricing the equal-weighted industry book-to-market portfolios. The observed three factor model root-mean-squared pricing error on the equal-weighted portfolios is eight basis points per month, compared to twenty basis points per month for the market model. In both cases a GRS test fails to reject the hypothesis that the pricing errors are jointly zero ($F_{5,982} = 1.46$, for a p-value $= 20.2\%$, and $F_{5,984} = 2.01$, for a p-value $= 7.5\%$, respectively).

Interestingly, the dispersion in HML loadings across industries exceeds those within industries despite the facts that 1) the dispersion in book-to-market within industries is approximately twice that observed across industries, and 2) the intra-industry variation in book-to-market is strongly associated with differences in expected returns while the variation in book-to-market across industries is not. This fact essentially guarantees the inefficiency of HML. The construction of HML ensures that the factor covaries positively with the returns to a portfolio long value industries and short growth industries. This variation, which can be hedged, is unpriced absent systematic variation in expected returns across industries, tautologically.

Table VII. Value-weighted results employing portfolios double sorted on book-to-market within and across industries

This table shows value-weighted average excess returns and book-to-market ratios (in parentheses) for portfolios double sorted on book-to-market within industries and industry book-to-market (left half of top panel). It also gives results of time-series regressions of both sorts' high-minus-low portfolios' returns on the Fama-French factors, with test-statistics [in square brackets]. The sample covers July 1926 to December 2008.

	Industry BM quintiles					Industry value strategies				
	L	2	3	4	H	r^e	α	β_{mkt}	β_{smb}	β_{hml}
Intra-industry BM quintiles										
L	0.48	0.57	0.58	0.61	0.50	0.02	−0.22	−0.07	0.08	0.65
	(0.24)	(0.33)	(0.44)	(0.52)	(0.68)	[0.16]	[−1.91]	[−3.35]	[2.14]	[20.4]
2	0.56	0.66	0.66	0.76	0.55	−0.01	−0.46	0.06	0.17	0.93
	(0.40)	(0.56)	(0.66)	(0.81)	(1.28)	[−0.06]	[−3.95]	[2.46]	[4.69]	[28.5]
3	0.57	0.65	0.74	0.73	0.70	0.13	−0.15	−0.08	−0.15	0.88
	(0.58)	(0.80)	(0.89)	(1.11)	(2.03)	[0.86]	[−1.31]	[−3.46]	[−4.32]	[27.8]
4	0.70	0.72	0.86	0.86	0.93	0.23	−0.21	0.11	−0.23	1.07
	(0.83)	(1.06)	(1.22)	(1.53)	(3.19)	[1.29]	[−1.64]	[4.40]	[−5.69]	[29.6]
H	0.71	0.83	0.85	1.06	1.11	0.40	−0.15	0.08	0.05	1.22
	(1.60)	(2.44)	(2.81)	(3.47)	(9.30)	[1.86]	[−0.91]	[2.38]	[0.98]	[26.7]
Intra-industry value strategies										
r^e	0.23	0.26	0.27	0.45	0.61					
	[1.85]	[1.75]	[1.88]	[3.05]	[3.34]					
α	0.04	−0.18	−0.11	0.04	0.11					
	[0.39]	[−1.68]	[−0.97]	[0.39]	[0.79]					
β_{mkt}	−0.01	0.11	0.08	0.11	0.14					
	[−0.33]	[5.07]	[3.63]	[4.72]	[5.02]					
β_{smb}	0.47	0.49	0.46	0.40	0.44					
	[13.1]	[14.5]	[13.0]	[11.2]	[9.78]					
β_{hml}	0.20	0.66	0.55	0.61	0.77					
	[6.26]	[21.3]	[17.2]	[18.9]	[18.9]					

These results suggest that a fundamental rethinking of the value premium is required. The value premium is not driven by industry variation. It is driven, as predicted by the model, by intra-industry variation in firms' production efficiencies.

3.2.2 Double Sorts on Intra-Industry Book-to-Market and Industry Book-to-Market

Similar results are obtained by independently double sorting stocks on intra-industry book-to-market and industry book-to-market. Each of the 25 portfolios resulting from the double sorting procedure have roughly the same number of firms, by construction. Tables VII and VIII present results of tests employing these portfolios, value and equal weighted respectively. These results are consistent with those of the univariate sorts presented in Table VI. While value firms generate higher returns than growth firms across industry book-to-market quintiles, value firms in value

Table VIII. Equal-weighted results employing portfolios double sorted on book-to-market within and across industries

This table shows equal-weighted average excess returns and book-to-market ratios (in parentheses) for portfolios double sorted on book-to-market within industries and industry book-to-market (left half of top panel). It also gives results of time-series regressions of both sorts' high-minus-low portfolios' returns on the Fama-French factors, with test-statistics [in square brackets]. The sample covers July 1926 to December 2008.

	Industry BM quintiles					Industry value strategies				
	L	2	3	4	H	r^e	α	β_{mkt}	β_{smb}	β_{hml}
Intra-industry BM quintiles										
L	0.37	0.35	0.55	0.62	0.56	0.19	−0.03	−0.14	0.01	0.75
	(0.26)	(0.33)	(0.40)	(0.49)	(0.65)	[1.39]	[−0.32]	[−6.71]	[0.20]	[24.8]
2	0.59	0.73	0.79	0.85	0.80	0.21	−0.04	−0.13	−0.14	0.88
	(0.46)	(0.60)	(0.69)	(0.85)	(1.23)	[1.51]	[−0.42]	[−6.64]	[−4.51]	[32.1]
3	0.80	0.90	0.91	0.92	0.99	0.19	−0.13	−0.08	−0.05	0.96
	(0.67)	(0.88)	(0.96)	(1.17)	(2.03)	[1.29]	[−1.29]	[−4.16]	[−1.62]	[32.7]
4	1.07	0.97	1.07	1.08	1.25	0.18	−0.16	−0.02	−0.09	0.93
	(1.00)	(1.27)	(1.41)	(1.67)	(3.61)	[1.19]	[−1.39]	[−0.97]	[−2.63]	[29.1]
H	1.42	1.33	1.44	1.51	1.39	−0.03	−0.28	−0.10	−0.05	0.79
	(5.54)	(7.09)	(7.27)	(11.6)	(32.8)	[−0.15]	[−1.76]	[−3.21]	[−0.95]	[17.8]
Intra-industry value strategies										
r^e	1.05	0.98	0.89	0.89	0.84					
	[6.28]	[6.18]	[5.23]	[5.48]	[4.38]					
α	0.56	0.52	0.36	0.40	0.32					
	[4.96]	[4.82]	[3.31]	[3.50]	[2.21]					
β_{mkt}	−0.02	−0.02	0.01	0.09	0.03					
	−0.83]	[−1.15]	[0.38]	[3.80]	[0.89]					
β_{smb}	0.63	0.50	0.61	0.38	0.58					
	18.0]	[15.0]	[17.9]	[10.6]	[12.9]					
β_{hml}	0.88	0.89	0.94	0.85	0.91					
	[27.5]	[29.2]	[30.5]	[25.8]	[22.4]					

industries do not produce higher returns than value firms in growth industries, and growth firms in value industries do not produce higher returns than growth firms in growth industries. The average return spread between the intra-industry value and growth firms across the industry book-to-market quintiles is 36 basis points per month (value-weighted) or 93 basis points per month (equal-weighted), and a GRS test rejects the hypothesis that these five strategies' expected returns are jointly zero ($F_{5,973} = 3.30$, for a p-value $= 0.6\%$, value-weighted; $F_{5,973} = 9.37$, for a p-value $= 0.0\%$, equal-weighted). The average return spread between value industries and growth industries across the intra-industry book-to-market quintiles is much smaller, fifteen basis points per month (both value and equal weighted), and a GRS test fails to reject the hypothesis that these five strategies' expected returns are jointly zero ($F_{5,973} = 1.17$, p-value $= 32.3\%$; $F_{5,973} = 0.78$, for a p-value $=$ 56.4%, equal-weighted).

While intra-industry value stocks (i.e., inefficient firms) yield higher returns than intra-industry growth stocks (i.e., efficient firms) across industry book-to-market quintiles, the three factor model helps price the intra-industry high-minus-low portfolios, improving the observed root-mean-squared pricing error on the five intra-industry value strategies from 39 to 11 basis points per month, value-weighted, and 93 to 44 basis points per month, equal-weighted. A GRS test also fails to reject that the pricing errors on the five value-weighted portfolios relative to the three factor model are jointly zero ($F_{5,970} = 1.05$, p-value $= 38.4\%$), though the test emphatically rejects that the pricing errors on the five equal-weighted portfolios relative to the three factor model are jointly zero ($F_{5,970} = 8.23$, p-value $= 0.0\%$).

In contrast, the three factor model performs worse than no model at all in explaining the industry high-minus-low portfolio returns. These portfolios generate only insignificant returns, but load heavily on HML, even more heavily than do the intra-industry value strategies. As a consequence, the inter-industry value strategies, both value-weighted and equal-weighted, all have negative Fama-French alphas, and the observed three factor root-mean-squared pricing errors are as large as the absolute root-mean-squared pricing errors, 26 versus 21 basis points per month value-weighted, and 16 versus 17 basis points per month equal-weighted. A GRS test strongly rejects that the value-weighted strategies pricing errors relative to the three factor model are jointly zero ($F_{5,970} = 3.28$, p-value $= 0.6\%$), though fails to reject the same hypothesis for the equal-weighted strategies ($F_{5,970} = 0.79$, p-value $= 55.8\%$).

4. Conclusion

This paper provides direct empirical evidence, previously absent in the literature, for the “operating leverage hypothesis,” which underlies most theoretical explanations of the value premium. I also identify the reason that direct evidence has been elusive: difficulties, both practical and theoretical, associated with testing the hypothesis directly.

I provide additional indirect support for the operating leverage hypothesis, by deriving and testing implications that do not require direct observation of operating leverage. I demonstrate that, consistent with the predictions of my dynamic equilibrium model, expected returns and book-to-market are strongly correlated within industries, but almost uncorrelated across industries.

My results have important implications for investors and researchers. Identifying and isolating the source of the the value premium– intra-industry as opposed to industry variation in book-to-market– allows for the construction of more efficient value strategies. This helps investors design more profitable trading strategies. It should also yield more accurate benchmarks, allowing for a more precise evaluation of the true performance of value strategies, and the managers that employ them.

A. Appendix: Model and Solution

A.1 ECONOMY

The industry consists of n competitive firms, which are assumed to maximize the expected present value of risk-adjusted cash flows discounted at the constant risk-free rate r. These firms employ capital, which may be brought into the industry at a price that I will normalize to one and may be sold outside the industry at a price $\alpha < 1$, in conjunction with non-capital inputs to produce a flow of a non-storable good or service.[4]

A firm can produce a flow of the industry good proportional to the level of capital it employs, and proportional to its firm-specific production efficiency. That is, at any time a firm "i" can produce a quantity ("supply") of the good $S_t^i = K_t^i / c_i$ where K_t^i is the firm's capital stock and c_i is the firm's capital requirement per unit of production (i.e., the firm's inverse capital productivity). Aggregate production is then $S_t = \Gamma \mathbf{K}_t$, where $\mathbf{K}_t = (K_t^1, K_t^2, \ldots, K_t^n)'$ and $\Gamma = (c_1^{-1}, c_2^{-1}, \ldots, c_n^{-1})$ denote the vectors of firms' capital stocks and firms' capital productivities, respectively. Aggregate capital employed in the industry is $K_t = 1\mathbf{K}_t$ where $1 = (1, 1, \ldots, 1)$ is the n-vector of ones.

In the goods market firms face a stochastic level of iso-elastic demand, with price elasticity $1/\gamma > 1/n$.[5] Operating revenue generated by each unit of capital employed by firm i is therefore P_t / c_i, where

$$P_t = \left(\frac{X_t}{S_t} \right)^{\gamma}, \tag{A1}$$

where I assume that the multiplicative demand shock X_t is a geometric Brownian process under the risk-neutral measure,

$$dX_t = \mu_X X_t dt + \sigma_X X_t dB_t$$

where $\mu_X < r$ and σ_X are known constants, and B_t is a standard Wiener process.

Production also entails a non-discretionary operating cost, assumed to be proportional to the level of capital employed, with a unit cost per period of η.[6] Firm i's operating profits may consequently be written, in terms of primatives and the

[4] In the case of complete irreversibility (i.e., $\alpha = 0$) I still allow for the free disposal of capital.

[5] This condition assures that no firm can increase its own revenues by decreasing output.

[6] The parameter η corresponds closely to the operating leverage measure employed in the empirical tests.

control, as

$$R^i(\mathbf{K}_t, X_t) = \frac{K_t^i}{c_i}\left(\frac{X_t}{\Gamma \mathbf{K}_t}\right)^{\gamma} - K_t^i \eta. \tag{A2}$$

Finally, each firm's capital stock changes over time due to investment, disinvestment and depreciation. At any time firms may acquire and deploy new capital within the industry, at a unit price equal to one, or sell capital that will be redeployed outside the industry, at the unit price $\alpha < 1$. Capital depreciates at a constant rate δ. The change in a firm's capital stock can therefore be written as $dK_t^i = -\delta K_t^i + dU_t^i - dL_t^i$, where U_t^i (respectively, L_t^i) denotes firm i's gross cumulative investment (respectively, disinvestment) up to time t.

A.2 FIRM'S OPTIMIZATION PROBLEM

The value of a firm's investment depends on the price of the industry good, and consequently on the aggregate level of capital employed in the industry. As a consequence, the value of a firm depends not only on how it invests, but also on how other firms invest. Moreover, because each firm's investment itself affects prices, any given firm's investment strategy affects the investment strategy employed by other firms.

Firms are assumed to maximize discounted cash flows, so the value of firm i is given by

$$\begin{aligned} V^i(\mathbf{K}_t, X_t) = \max_{\{dU_{t+s}^i, dL_{t+s}^i\}} \mathbf{E}_t\Bigg[\int_0^{\infty} e^{-rs}\big(R_i(\mathbf{K}_{t+s}, X_{t+s})ds \\ - dU_{t+s}^i + \alpha dL_{t+s}^i\big)\Big|\big\{dU_{t+s}^{-i}, dL_{t+s}^{-i}\big\}\Bigg] \end{aligned} \tag{A3}$$

where $\{dU_t^{-i}, dL_t^{-i}\}$ is used to denote other firms' investment/disinvestment at time t, and the expectation is with respect to the risk-neutral measure.

A.3 EQUILIBRIUM

I am interested in exhibiting a simple equilibrium, and therefore restrict my attention to Nash-Cournot strategies.[7] Equilibrium issues, and a Markov perfect equilibrium that support the same long run equilibrium path as that presented here, are discussed in Novy-Marx (2009b).

Proposition A.1. *Suppose that the current state of the economy satisfies the following "long history" conditions:*

[7] Consideration of collusive strategies, such as that employed by Green and Porter (1984), while extremely interesting, is beyond the scope of this analysis.

1. *The long run participation constraint: each firm's production is "sufficiently efficient," in that its capital requirement per unit of production is not too high, satisfying*

$$c_i < c_{\max} \equiv \frac{\overline{c}}{1 - \frac{\gamma}{n}} \tag{A4}$$

where $\overline{c} \equiv \frac{1}{n}\sum_{j=1}^{n} c_j$ *is the equal-weighted industry average capital requirement per unit of production.*[8]

2. *The long run industry organization: firms produce in proportion to their "cost wedges,"* $S_t^i/S_t^j = (c_{\max} - c_i)/(c_{\max} - c_j)$*, which requires that firms' capital stocks satisfy*

$$K_0^i = \left(\frac{\overline{c}c_i - \left(1 - \frac{\gamma}{n}\right)c_i^2}{\overline{c}^2 - \left(1 - \frac{\gamma}{n}\right)\overline{c^2}}\right)\frac{K_0}{n} \tag{A5}$$

for each i, where $\overline{c^2} \equiv \frac{1}{n}\sum_{i=1}^{n} c_j^2$.

Then the Nash-Cournot investment strategy for firm j consists of the investment/disinvestment plan

$$dU_t^j - dL_t^j = \gamma^{-1} K_t^j \, dM_t \tag{A6}$$

where $M_t = M_{t-T_i}^i$ for $t \in [T_i, T_{i+1})$,[9]

$$M_s^i = (-1)^i \max_{s' \in [0,s]} \left\{(-1)^i y_{s'}^i, \ln\left(P_U/P_0\right) \mathbb{1}_{i=0}\right\} \tag{A7}$$

$$y_s^i = y_{s+T_i} - y_{T_i}, \tag{A8}$$

$y_t = \gamma \ln(e^{\delta t} X_t/X_0)$, $T_0 = 0$, $T_{i+1} = T_i + \min_{s>0}\{|y_s^i - M_s^i| = \ln(P_U/P_L)\}$,

$$P_U = \frac{(1+\psi)c_{\max}}{\Pi(\zeta^{-1})} \tag{A9}$$

$$P_L = \frac{(\alpha+\psi)c_{\max}}{\Pi(\zeta)}, \tag{A10}$$

$\psi = \frac{\eta}{r+\delta}$ *is the capitalized flow costs associated with operating a unit of capital in perpetuity,* $\Pi(\zeta^{-1})$ *and* $\Pi(\zeta)$ *are the perpetuity factors for the equilibrium price*

[8] This first condition is satisfied trivially if, given the order set of firms' unit costs $c_1 \leq c_2 \leq \ldots \leq c_M$, I let $n \equiv \max\{i \in \{1, \ldots, M\} | \, c_i < \frac{\overline{c}_i}{1-\frac{\gamma}{i}}\}$ where $\overline{c}_i = \frac{1}{i}\sum_{j=1}^{i} c_j$ and restrict attention to the first n firms.

[9] Because cumulative investment and disinvestment are non-decreasing processes, the increments of these processes correspond to the positive and negative increments of dM_t, respectively.

process at the investment and disinvestment thresholds P_U and P_L, respectively,

$$\Pi(x) = \left(1 - \frac{\sigma^2}{2(r+\delta)}\left(\frac{y'(1) - y'(x)}{y(x)}\right)x\right)\pi \tag{A11}$$

where $y(x) = x^{\beta_p} - x^{\beta_n}$, $\beta_p + \beta_n = -2(\frac{\mu}{\sigma^2} - \frac{1}{2})$ and $\beta_p - \beta_n = 2\sqrt{(\frac{\mu}{\sigma^2} - \frac{1}{2})^2 + \frac{2(r+\delta)}{\sigma^2}}$ for $\mu = \gamma(\mu_X + \delta + (\gamma - 1)\frac{\sigma_X^2}{2})$ and $\sigma = \gamma\sigma_X$, $\pi = \frac{1}{r+\delta-\mu}$, *and* $\zeta = P_U/P_L > 1$ *is uniquely determined by*

$$\frac{\Pi(\zeta)}{\zeta\Pi(\zeta^{-1})} = \frac{\alpha + \psi}{1 + \psi}. \tag{A12}$$

Note that under this strategy all firms invest in new capital, in proportion to their existing capital stock, whenever the goods price P_t reaches P_U, and disinvest whenever P_t reaches P_L. To see this, remember that $P = (X/S)^\gamma$, so

$$d\ln P_t = \gamma\left(d\ln X_t + \delta dt - \frac{\sum_j c_j^{-1}\left(dU_t^j - dL_t^j\right)}{\sum_j c_j^{-1}K_t^j}\right), \tag{A13}$$

which under the equilibrium strategy reduces to $d\ln P_t = \gamma(d\ln X_t + \delta dt) - dM_t$. Integrating subject to the initial conditions yields

$$\ln P_t = a_t + z_t - M_t \tag{A14}$$

where if $t \in [T_i, T_{i+1})$ then $a_t = \ln P_U$ if i is odd and $a_t = \ln P_L$ otherwise, $z_t = y^i_{t-T_i}$, and M_t is given in Proposition A.1. This describes a doubly reflected geometric Brownian goods price processes, with firms investing at the upper reflecting barrier P_U and disinvesting at the lower barrier P_L.

Proof of the proposition: The Bellman equation corresponding to firm i's optimization problem (Equation (A3)) is

$$\begin{aligned} rV^i(\mathbf{K}, X) = {} & R^i(\mathbf{K}, X) - \delta\,\mathbf{K}\cdot\nabla_{\mathbf{K}}V^i(\mathbf{K}, X) \\ & + \mu_X X V_X^i(\mathbf{K}, X) + \frac{1}{2}\sigma_X^2 X^2 V_{XX}^i(\mathbf{K}, X). \end{aligned} \tag{A15}$$

This equation essentially demands that the required return on the firm at each instant equals the expected risk-adjusted return (cash flows and capital gains). It holds identically in K_i, so taking partial derivatives of the left and right hand sides with respect to K_i yields

$$\begin{aligned} (r+\delta)V_{K^i}^i(\mathbf{K}, X) = {} & R_{K^i}^i(\mathbf{K}, X) - \delta\,\mathbf{K}\cdot\nabla_{\mathbf{K}}V_{K^i}^i(\mathbf{K}, X) \\ & + \mu_X X V_{XK^i}^i(\mathbf{K}, X) + \frac{1}{2}\sigma_X^2 X^2 V_{XXK^i}^i(\mathbf{K}, X). \end{aligned} \tag{A16}$$

Then using that $V^i(\mathbf{K}, X)$ is homogeneous degree one in $\mathbf{K}$ and in X, so $q_i(\mathbf{K}, X) \equiv V^i_{K_i}(\mathbf{K}, X)$ is homogeneous degree zero in $\mathbf{K}$ and X, and that $\mu = \gamma(\mu_X + \delta + (\gamma - 1)\sigma_X^2/2)$ and $\sigma = \gamma\sigma_X$, I can rewrite the previous equation as

$$(r+\delta)q_i(P) = \left(\frac{1-\frac{\gamma}{n}}{\bar{c}}\right)P + \mu P q_i'(P) + \frac{1}{2}\sigma^2 P^2 q_i''(P) - \eta. \tag{A17}$$

Using the "myopic strategy" solution technique of Leahy (1993), I can "guess" that the firm's marginal valuation of capital is the product of 1) its marginal revenue products of capital and 2) the unit value of revenues given the equilibrium price process. That is, I will guess that $q_i(\mathbf{K}_t, X_t) \equiv V^i_{K_i}(\mathbf{K}_t, X_t)$ may be written as

$$q_i(K_t^i, P_t) = \left(R^i_{K_i}\left(K_t^i, P_t\right) + \eta\right)\pi(P_t) - \psi \tag{A18}$$

where $R^i(K_t^i, P_t) = K_t^i(\frac{P_t}{c_i} - \eta)$ is the firm's unit profits, $\pi(P_t) = \mathbf{E}[\int_0^\infty e^{-(r+\delta)s}\frac{P_{t+s}}{P_t}ds]$ is the unit value of revenue for the geometric Brownian price process reflected above at P_U and below at P_L and is given (see e.g., Novy-Marx (2009a)) by

$$\pi(P) = \pi P + \left(\frac{y\left(\frac{P}{P_U}\right)}{y\left(\frac{P_L}{P_U}\right)}\right)\left(\Pi\left(\frac{P_U}{P_L}\right) - \pi\right)\left(\frac{P_L}{P}\right) + \left(\frac{y\left(\frac{P}{P_L}\right)}{y\left(\frac{P_U}{P_L}\right)}\right)\left(\Pi\left(\frac{P_L}{P_U}\right) - \pi\right)\left(\frac{P_U}{P}\right). \tag{A19}$$

The firm's revenue depends on its capital stock directly, because it uses the capital stock to produce the revenue generating good, and indirectly, because the price of the industry good depends, partly, on the firm's production. The firm's marginal revenue product of capital, differentiating firm revenue $R^i(\mathbf{K}_t, X_t) = K_t^i P_t/c_i$ with respect to K^i, is

$$R^i_{K^i}\left(K_t^i, P_t\right) + \eta = c_i^{-1}P_t + c_i^{-1}K_t^i\frac{dP_t}{dK_t^i}. \tag{A20}$$

Substituting the previous equation into Equation (A18), and using the facts that $\frac{dP_t}{dK_t^i} = -\gamma\frac{P_t}{c_i S_t}$, which comes from differentiating the inverse demand function $P_t = X_t^\gamma S_t^{-\gamma}$ with respect to K_t^i, and $S_t^i/S_t = (\bar{c} - (1-\frac{\gamma}{n})c_j)/\gamma\bar{c}$, which follows from the second condition of Proposition A.1, I have that

$$q(P_t) = c_{\max}^{-1}P_t\,\pi(P_t) - \psi, \tag{A21}$$

where explicit dependence on i has been dropped because firms' marginal valuations of capital equate.

It is then simple to check that $q(P)$ satisfies the differential equation associated with the Bellman equation. The marginal value of capital given in the previous equation satisfies Equation (A17) if and only if

$$(r+\delta)P\pi(P) = P + \mu P\frac{d}{dP}(P\pi(P)) + \frac{1}{2}\sigma^2 P^2\frac{d^2}{dP^2}(P\pi(P)). \tag{A22}$$

This must hold for all P, so using the fact that

$$P\pi(P) = \pi P + aP^{\beta_n} + bP^{\beta_p} \tag{A23}$$

for some a and b (see, for example, Equation (A19)) and, matching terms of equal P-orders on the left and right hand sides of Equation (A22), I then have that Equation (A17) holds if and only if

$$(r+\delta-\mu)\pi = 1$$
$$(r+\delta) - \left(\mu - \frac{\sigma^2}{2}\right)\beta_n - \frac{\sigma^2}{2}\beta_n^2 = 0$$
$$(r+\delta) - \left(\mu - \frac{\sigma^2}{2}\right)\beta_p - \frac{\sigma^2}{2}\beta_p^2 = 0,$$

which is easily verified.

The shadow price of capital, $q(P)$, also satisfies the necessary boundary conditions. Evaluating Equation (A21) at P_U and P_L yields

$$q(P_U) = 1 \tag{A24}$$

$$q(P_L) = \alpha. \tag{A25}$$

The smooth pasting condition at both boundaries, i.e., that $q_i'(P_U) = q_i'(P_L) = 0$, follows immediately from Equation (A21) and the construction of $\pi(P)$.

A.4 VALUATION

Proposition A.2. *Average-Q for firm i, as a function of the price of the industry good, is given by*

$$Q_t^i = q(P_t) + \theta_i\left((q(P_t)+\psi) + a_n\left(\frac{P_t}{P_L}\right)^{\beta_n} + a_p\left(\frac{P_t}{P_U}\right)^{\beta_p}\right) \tag{A26}$$

where $\theta_i = \frac{c_{\max}}{c_i} - 1$ *is firm i's "excess productivity,"*

$$a_n = -\frac{(1+\psi) - \zeta^{\beta_p}(\alpha+\psi)}{(\gamma\beta_n - 1)y(\zeta)} \tag{A27}$$

$$a_p = \frac{(\alpha+\psi) - \zeta^{-\beta_n}(1+\psi)}{(\gamma\beta_p - 1)y(\zeta^{-1})}, \tag{A28}$$

and the other parameters agree with those given in Proposition A.1.

Proof of the proposition: The value of deployed capital is the expected discounted value of the operating profits it generates given the equilibrium goods price process,

$$\hat{V}^i\left(P_t, K_t^i\right) = \left(\frac{K_t^i}{c_i}\right) P_t \pi(P_t) - K_t^i \left(\frac{\eta}{r+\delta}\right). \tag{A29}$$

Substituting Equation (A21), the equilibrium condition on marginal-q, into the previous equation, I get that a firm's average-Q of assets-in-place is affine in its marginal-q, given by

$$\frac{\hat{V}_t^i}{K_t^i} = q_t + \theta_i \left(q_t + \psi\right) \tag{A30}$$

where $\theta_i = \frac{c_{\max}}{c_i} - 1$. The first term in the previous equation is just the shadow price of capital, and is bounded between α and one, while the second term represents the capitalized value of rents expected to accrue to the deployed capital.

Total firm value also includes rents expected to accrue to future capital deployments, which will be bought at a price below the value of the revenues it is expected to generate. It also accounts for the costs associated with reducing capacity to support prices in "bad times," when capital will be sold at a price below the revenues it could have been expected to generate.

Firm i's value satisfies the standard differential equation, $\mu P V_P + \frac{\sigma^2}{2} P^2 V_{PP} = (r+\delta)V$, which implies

$$Q_t^i = \frac{\hat{V}_t^i}{K_t^i} + a_n^i \left(\frac{P_t}{P_L}\right)^{\beta_n} + a_p^i \left(\frac{P_t}{P_U}\right)^{\beta_p} \tag{A31}$$

for some a_n^i and a_p^i. This, taken with the differentiability of firm value at the investment and disinvestment boundaries, implies the following proposition.

Capacity is insensitive to changes in the multiplicative demand shock away from the boundary, so

$$\begin{aligned} \left.\frac{dV_i}{dX}\right|_{X=X_U^-} &= K_i \left(\frac{dP}{dX}\right) \frac{d}{dP} \left.\left(\frac{\hat{V}^i}{K^i} + a_n^i \left(\frac{P}{P_L}\right)^{\beta_n} + a_p^i \left(\frac{P}{P_U}\right)^{\beta_p}\right)\right|_{X=X_U^-} \\ &= \frac{\gamma K_i}{X^*} \left(\beta_n a_n^i \zeta^{\beta_n} + \beta_p a_p^i\right) \end{aligned} \tag{A32}$$

where I have used the facts that value of deployed capital is insensitive to changes in X at the development boundary and $\frac{dP}{dX} = \gamma P / X$.

At the boundary, homogeneity of the value function implies $\frac{d}{dX} \frac{V_i}{K_i} |_{X=X_U^+} = 0$, and the supply response ensures the price never exceeds P_U so $\frac{d \ln K_i}{d \ln X} |_{X=X_U^+} = 1$, so

$$\left.\frac{d\,(V_i - K_i)}{dX}\right|_{X=X_U^+} = \left(\frac{V_i}{K_i} - 1\right)\left.\frac{dK_i}{dX}\right|_{X=X_U^+} = \frac{V_i^* - K_i}{X^*}. \tag{A33}$$

The value function is differentiable at the boundary, $\frac{d}{dX}V_i|_{X=X_U^-} = \frac{d}{dX}(V_i - K_i)|_{X=X_U^+}$, which, using the results of the previous two equations, yields

$$\gamma\left(\beta_n a_n^i \zeta^{\beta_n} + \beta_p a_p^i\right) = Q_U^i - 1, \tag{A34}$$

or, rearranging using the fact that at the investment boundary $\hat{V}_U^i/K^i = 1 + \theta_i(1 + \psi)$ where $\theta_i = c_{\max}/c_i - 1$, that

$$(\gamma\beta_n - 1)\,a_n^i \zeta^{\beta_n} + (\gamma\beta_p - 1)a_p^i = \theta_i(1 + \psi). \tag{A35}$$

A completely analogous calculation at the disinvestment boundary implies

$$(\gamma\beta_n - 1)\,a_n^i + (\gamma\beta_p - 1)a_p^i \zeta^{-\beta_p} = \theta_i(\alpha + \psi). \tag{A36}$$

Solving the previous two equations simultaneously yields $a_n = a_n^i/\theta_i$ and $a_p = a_p^i/\theta_i$. ■

Note that Equation (A26) implies that less efficient firms have higher levels of operating leverage. Operating leverage depends on the ratio of the capitalized costs associated with operating capital in perpetuity and the capitalized value of the profits the capital is expected to generate (Equation (2)). This ratio is decreasing in firm efficiency, because while my model assumes that the numerator is homogenous within an industry, the denominator is increasing in firms' "excess productivities." The denominator is, using Equation (A26), the shadow price of capital plus the capitalized rents to deployed capital. These rents are proportional to $\theta_i = \frac{c_{\max}}{c_i} - 1$, implying higher operating leverage for less efficient firms. That is, less efficient firms are more "levered," in the operating leverage sense, which results in these firms being more exposed to the economic risks underlying the economy.

A.5 EXPECTED RETURNS

Equation (A26), which specifies average-Q as a function of firm and industry characteristics, can be used to explicitly calculate the sensitivity of firm value to demand, providing a means to study the relation between market-to-book and expected returns. The following proposition relates firms' risk factor loadings, and consequently their expected rates of return, to prices in the goods market. In conjunction with the previous proposition this will allow us to explicitly relate firms' expected returns to their book-to-markets.

Proposition A.3. *The expected excess rate of return to firm i is* $\beta_t^i \lambda_t$ *where* λ_t *is the time-t price of exposure to the priced risk factor (X) and*

$$\beta_t^i = \left(Q_t^i\right)^{-1} \left(\frac{\pi P_t}{c_i} + C_{\beta_n}^i \left(\frac{P_t}{P_L}\right)^{\beta_n} \beta_n + C_{\beta_p}^i \left(\frac{P_t}{P_U}\right)^{\beta_p} \beta_p \right) \tag{A37}$$

where

$$C_{\beta_n}^i = \left(\gamma \beta_n \left(\frac{c_{\max}}{c_i}\right) - 1\right) a_n - \left(\frac{\pi P_L}{c_i}\right)\left(\frac{\zeta^{\beta_p} - \zeta}{y(\zeta)}\right) \tag{A38}$$

$$C_{\beta_p}^i = \left(\gamma \beta_p \left(\frac{c_{\max}}{c_i}\right) - 1\right) a_p - \left(\frac{\pi P_U}{c_i}\right)\left(\frac{\zeta^{-1} - \zeta^{-\beta_n}}{y(\zeta^{-1})}\right). \tag{A39}$$

Proof of the proposition: Combining the valuation Equation (equations (A26)) with the explicit characterization of marginal-q (equations (A21) and (A19)), together with the fact that $\Pi(\zeta)P_L = (\alpha + \psi)c_{\max}$ and $\Pi(\zeta^{-1})P_U = (1 + \psi)c_{\max}$, I have that a firm's average-Q is given by

$$Q_t^i = \frac{\pi P_t}{c_i} + C_{\beta_n}^i \left(\frac{P_t}{P_L}\right)^{\beta_n} + C_{\beta_p}^i \left(\frac{P_t}{P_U}\right)^{\beta_p} - \psi. \tag{A40}$$

The proposition then follows directly. ■

In normal times inefficient producers are more exposed to the underlying risks in the economy, because the exposure of their revenues to the risk factor is levered more by their high production costs. In good times, however, they are relatively insulated from these risks, which are largely absorbed by the capacity response resulting from firms' competitive investment decisions. At these times, efficient producers still benefit from positive economic shocks. In response to these shocks they expand, buying capital at a price that is lower than its average value to the firm, and capturing the expected surplus.

A.6 UNCONDITIONAL EXPECTED RETURNS

While Equation (A37) specifies firms' conditional expected rates of returns, it is now simple to characterize firms' unconditional expected rates of return. A firm's unconditional expected excess rate of return is the average price of risk scaled by the firm's exposure to the risk factor. This is given explicitly in the following proposition.

Proposition A.4. *The unconditional expected rate of return to firm i is*

$$r_i^e = \int_{P_L}^{P_U} \beta^i(p)\lambda(p)d\nu(p) \tag{A41}$$

where $\beta^i(P_t)$ *is given in Proposition A.3 and* $d\nu(p)$*, the stationary density for the risk-neutral price process, is given by*

$$d\nu(p) = \phi\left(\frac{p^{\phi-1}}{P_U^{\phi} - P_L^{\phi}}\right)dp \tag{A42}$$

for $\phi = \frac{2\mu}{\sigma^2} - 1$.

Proof of the proposition: Suppose X_t is a geometric Brownian process with drift μ and volatility σ, and a lower reflecting barrier at l and an upper reflecting barrier at 1. Then

$$\lim_{t\to\infty} t^{-1}\mathbf{E}\left[\int_0^t \mathbb{1}_{[l,z]}X_s ds\right] = \mathbf{P}[X < z] \tag{A43}$$

where $\mathbb{1}_A(\omega) = 1$ if $\omega \in A$ and $\mathbb{1}_A(\omega) = 0$ otherwise, and X is has the stationary distribution of the process X_t.

If Y_t is a geometric Brownian process with the same drift and volatility, also reflected above at 1 but unreflected below, then by the Markovian nature of the processes

$$X_t \overset{\mathrm{d}}{=} Y_{s_t} \tag{A44}$$

where $\overset{\mathrm{d}}{=}$ denotes "equal in distribution," and $s_t \equiv \min\{s | \int_0^s \mathbb{1}_{[l,1]}Y_s ds = t\}$. So

$$\mathbf{P}[X < z] = \mathbf{P}[Y < z | Y > l] \tag{A45}$$

$$= \frac{\mathbf{P}[Y < z] - \mathbf{P}[Y < l]}{1 - \mathbf{P}[Y < l]}. \tag{A46}$$

Finally, using the fact that a Brownian process with positive drift reflected from above at zero has a stationary distribution that is exponentially distributed, with exponent equal to its drift divided by half its volatility squared, I have that $\mathbf{P}[Y < y] = y^{\phi}$ where $\phi = (\mu - \sigma^2/2)/(\sigma^2/2)$. Substituting this into the previous equation, and letting $X_t = P_t/P_U$, $l = P_L/P_U$ and $z = P_T/P_U$, yields the stationary distribution for the equilibrium price process,

$$\mathbf{P}[P < p] = \frac{p^{\phi} - P_L^{\phi}}{P_U^{\phi} - P_L^{\phi}}. \tag{A47}$$

Differentiating with respect to p yields the stationary density for the risk-neutral price process, and the proposition then follows directly. ■

Figure 1 is constructed by integrating book-to-market and conditional expected returns over this stationary density, while 1) varying the capital requirement per unit of production (c_i) while holding the industry operating cost parameter η fixed at two and a half, one and two fifths; and 2) varying the industry operating cost parameter while holding the capital requirement per unit of production fixed at its industry average. Other parameters employed are $r = 0.05$, $\mu = 0.03$, $\sigma = 0.20$, $\delta = 0.02$, $\gamma = 1$, $H = 0.02$, $\alpha = 0.25$, and $\lambda = 0.05$.

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